INVARIANT LASER BEAMS - FUNDAMENTAL PROPERTIES AND THEIR INVESTIGATION BY COMPUTER SIMULATION AND OPTICAL EXPERIMENT

Vladimir S. Pavelyev, Michael Duparré[*], Barbara Luedge[a], Victor A. Soifer, Richard Kowarschik[*] & Dimitriy L. Golovashkin

Image Processing Systems Institute, Molodogvardejskaya 151,Samara 443001, Russia

[*)] University of Jena, Institute of Applied Optics, Froebelstieg 1,D-07743 Jena, Fed. Rep. Germany

Abstract

Laser light modes are beams in whose cross-section the complex amplitude is described by *eigenfunctions* of the operator of light propagation in the waveguide medium. The fundamental properties of modes are their orthogonality and their ability to retain their structure during propagation for example in a lenslike medium, in free space or a Fourier stage. Novel Diffractive Optical Elements (DOEs) of MODAN-type [1] open up new promising potentialities of solving the tasks of generation, transformation, superposition and subsequent separation again of different laser modes. Now we present new results obtained by synthesis and investigation of beams consisting of more than one two-dimensional Gaussian laser modes with the same value of propagation constant (*invariant multimode beams)* formed by DOEs. The exploitation of these phenomena could enhance the fiber optical system transfer capacity without pulse enlargement.

1. Introduction

Laser light modes are beams in whose cross-section the complex amplitude is described by *eigenfunctions* of the operator of light propagation in the waveguide medium. The fundamental properties of modes are the property of retaining their structure and orthogonality during the propagation in a waveguide medium (for example free space or *lens-like waveguide*). Novel Diffractive Optical Elements (DOEs) of MODAN-type [1] open up new promising possibilities of solving the tasks of generation, transformation, superposition and subsequent separation again of different laser modes. In [2,3] we presented a MODAN, capable to transform a Gaussian (0,0) input beam into an unimodal Gauss-Hermite GH(1,0) complex amplitude distribution with high efficiency. Now we present new results, obtained by synthesis and investigation of beams, consisting of more than one transversal laser modes with the same value of propagation constant (invariant beams) and formed by DOEs. It's important to note, that invariant beams, excited by phase DOEs with high efficiency, can be used for optical communication purposes because of the absence of pulse enlargement phenomena [4,5]. We present theoretical as well as first experimental investigations of invariant beam propagation through Fourier stage and in the free-space. The results demonstrate promising perspectives for the selected concept in future.

2. Basic formalism

Let us define modes as light beams, which reproduce themselves during their propagation in a waveguide medium. Modal beams do not change their spatial structure in a proper waveguide medium. Every mode gets its own attenuation and its own phase delay, proportional to the optical path length and to the *propagation constant*. Thus, in the case of a graded-index optical fiber, the phase delay is continuously accumulated during the mode propagation. Furthermore, in this case guided modes reproduce their modal configuration after each path of sufficient length in the fiber. In this paragraph we discuss the mode properties, which are essential with respect to computer generation of optical elements matched to modes' complex amplitudes (MODANs). Let us set the Cartesian coordinates $(x, y, z) = (\mathbf{x}, z)$ in the medium of beam propagation. The two-dimensional vector $\mathbf{x} = (x, y)$ represents the transverse coordinates; z is the longitudinal coordinate along the optical axis. Guided modes under consideration are thought as located within the domain $\mathbf{x} \in G$ in the beam cross-section. We use the scalar representation of light field and the scalar diffraction theory without any consideration of polarization effects [4]. Thus, we describe the monochromatic or quasi-monochromatic field by the complex amplitude $w(\mathbf{x}, z)$ with wavelength λ or wavenumber k. Besides, by default we will suggest that waveguide medium (gradient index waveguide or free space) under consideration has a *translation invariant feature,* e.g. its characteristics do not change along the z - axis. Let's consider the Helmholtz equation for gradient index medium description

$$\nabla_\perp^2 w(x, y, z) + \frac{\partial^2 w(x, y, z)}{\partial z^2} + n^2(x, y) \cdot k^2 \cdot w(x, y, z) = 0 \qquad (1)$$

with the initial values $w|_{z=0} = w(x, y, 0)$ of the complex amplitude, where $\nabla_\perp = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$ is the transverse differential Hamiltonian operator. If finite diameter of waveguide is taken into account, then certain additional boundary conditions appear at the interface between core and cladding. According to [6], the modes of a graded-index optical fiber (Fig. 1) have a plane wavefront and obey equation

$$\nabla_\perp^2 \psi_{pl}(x, y) + \left[k^2 n^2(x, y) - \beta_{pl}^2\right] \cdot \psi_{pl}(x, y) = 0 . \qquad (2)$$

For any given distance z we have

$$w(x, y, z) = \gamma_{pl} \cdot \psi_{pl}(x, y), \qquad (3)$$

$$\gamma_{pl} = \exp\left[\left(i\beta_{pl} + \alpha_{pl}\right) \cdot z\right] \qquad (4)$$

where $\beta_{\mathbf{p}l}$ is the propagation constant and $\alpha_{\mathbf{p}l}$ is the coefficient of attenuation for the mode $\psi_{\mathbf{p}l}$. Thus, the modes of graded-index optical fiber satisfy the eigenvalue equation (2) for any distance z passed by light. Eigenvalues are specified by Eq. (4). It must be noted, that Eq. (3) describes the modal self-reproduction, that occurs with a constant scale regarding the Cartesian coordinates (*x*,*y*). For the propagation in a fiber with quadratic refractive index described in the form

$$n^2(r) = n_1^2\left(1 - 2\Delta\frac{r^2}{a^2}\right), \tag{5}$$

we have

$$\gamma_{pl}(z) = \exp(i\beta_{pl}z), \tag{6}$$

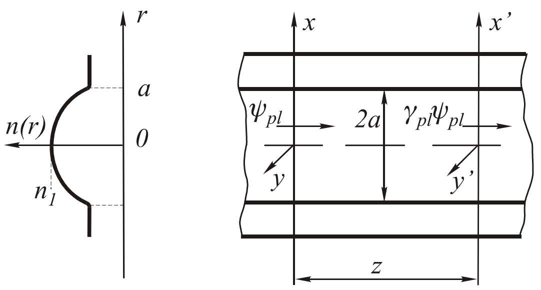

Fig. 1: Modes in a graded-index optical fiber with parabolic profile

Gauss-Laguerre- and Gauss-Hermite modes are eigenfunctions of the propagation operator in a waveguide medium with parabolic profile according to Eq. (5).

In polar coordinate system we obtain as solutions of the Helmholtz equation the well-known formulae for the complex amplitude of Gauss-Laguerre modes described in [1]

$$\psi_{pl}(\mathbf{x}) = E_{pl}\left(\frac{|\mathbf{x}|\sqrt{2}}{\sigma}\right)^l L_p^l\left(\frac{2\mathbf{x}^2}{\sigma^2}\right) * \\ * \exp\left(-\frac{\mathbf{x}^2}{\sigma^2}\right)\exp(\pm il\alpha) \tag{7}$$

where α is the polar angle of the vector (x, y) with an absolute $r = \sqrt{x^2 + y^2}$, L_p^l are the generalized Laguerre polynoms with $p, l = 0,1,2,...$ described in [8]. Here

$$E_{pl} = \frac{2}{\sigma\sqrt{2\pi \cdot l!C_{p+l}^l}} \tag{8}$$

is the normalizing constant. In some cases, the modes in Eq. (7) are written in a real form including $\sin(l\alpha)$ and $\cos(l\alpha)$ instead of $\exp(\pm il\alpha)$. As solutions of the Helmholtz equation for a waveguide with parabolic profile in the Cartesian coordinate system we obtain the well-known formulae for the complex amplitude of Gauss-Hermite “GH” modes described in [7],

$$\psi_{pl}(x, y) = E_{pl} \cdot H_p\left(\frac{\sqrt{2}x}{\sigma}\right) \cdot \\ \cdot H_l\left(\frac{\sqrt{2}y}{\sigma}\right) \cdot \exp\left[-\frac{x^2 + y^2}{\sigma^2}\right], \tag{9}$$

where $H_p(.)$ - is the Hermite polynom of p th order, σ is the mode fundamental radius and

$$E_{pl} = \frac{1}{\sigma} \cdot \sqrt{\frac{2}{\pi \cdot 2^{p+l} \cdot p!\cdot l!}} \tag{10}$$

is a normalization constant again. The propagation constant β_{pl} for Gaussian modes [1] is

$$\beta_{pl} = \sqrt{k^2 n_1^2 - \frac{4}{\sigma^2}\left(r_{pl} + 1\right)}, \tag{11}$$

where the two-dimensional integral mode number follows $r_{pl} = p+l$ for Gauss-Hermite- and $r_{pl} = 2p+l$ for Gauss-Laguerre modes, respectively. Let us summarize the well-known mode fundamental features [1]. Being natural (normal) or eigen-oscillations, the modes of a waveguide may be characterized by the following invariant and optimal properties:

1. *Each waveguide medium can be characterized by a* <u>*discrete*</u> *set* $\{\psi_{pl}(x, y)\}$ *of its eigen-oscillations - modes.*
2. *Modes are the unique two-dimensional base-functions that conserve the orthogonality during guided propagation in their native waveguide media.*
3. *Modes are the unique two-dimensional base-functions that conserve amplitude-phase structure during guided propagation in their native waveguide media.*
4. *Gaussian mode beams manifest a remarkable property of conservation of their type, while propagating not only in free-space but through lenses too. According to [7] Gaussian modes propagating through an optical Fourier-stage with the focal length* f_0, *change their parameters* σ *and* R *into* σ_F *and* R_F.

$$\sigma_F = \frac{\lambda f_0}{\pi\sigma}, \quad R_F = \infty \tag{12}$$

Phase of the beam changes in this case by the following value:

$$k f_0 + \arg(i^{2p+l}) = kf_0 + r_{pl}\frac{\pi}{2} \tag{13}$$

Note, that for two-dimensional waveguide cross-section the same particular value of propagation constant set $\{\beta_{pl}\}$ can correspond to more than one different modal functions: It is easy to see that two Gauss-Hermite modes with numbers (*p*,*l*) and (*l*,*p*) will have the same value $r_{pl} = p + l$ .In general, by virtue of Eqs. (3-6) and Eq. (11), under condition of $a_{pl} = 0$ (no attenuation) the propagation of any linear combination $\chi_{r_{pl}}(x, y)$ of more than one different Gaussian modes $\psi_{pl}(x, y)$ each with the same value of propagation constant β_{pl}

$$\chi_{r_{pl}}(x, y) = \sum_{r_{pl}=const} \tilde{C}_{pl}\psi_{pl}(x, y) \tag{14}$$

would be similar to propagation of isolated modes. So, a beam with a cross section corresponding to Eq. (14) will have no change in its amplitude-phase structure during propagation in waveguide medium. Amplitude-phase distributions having cross-section which can be described by Eq. (14) we will call *invariant multimode beams*. Invariant multimode beam of a waveguide may be characterized by the following invariance properties:

1. *Each discrete waveguide mode set* $\{\psi_{pl}(x,y)\}$ *is able to generate a continuous set of invariant multimode beams (!) because of the continuous character of complex-valued coefficients* $\tilde{C}_{pl}$ *in Eq.* (14).
2. *Self-reproduction: invariant multimode beam does not change its amplitude-phase structure and size during propagation in a waveguide medium.*
3. *Gaussian multimode invariant beam does not change its amplitude-phase structure during propagation in free-space.*
4. *Gaussian multimode invariant beam does not change its amplitude-phase structure during propagation through a Fourier-stage, whereas the fundamental radius or self-mode parameter changes as in Eq.* (12).
5. *An invariant beam can propagate through a waveguide without pulse enlargement effect* [5].
6. *Two different invariant beams with different value* r_{pl} *are orthogonal under condition that* $\left|\tilde{C}_{pl}\right| = const$ *for all* (p,l) *pairs.*

3. Invariant multimode beam investigation by methods of computational and optical experiments.

In [2,3] we presented a MODAN capable of transforming a Gaussian (0,0) input beam into a unimodal Gauss-Hermite (GH) (1,0) complex amplitude distribution with high efficiency. In order to demonstrate fundamental properties of invariant multimodal beams, we designed a MODAN which should be able to transform one single transversal mode into the sum of two other modes with uniform propagation constant. For the input beam we selected the Gauss (0,0) mode characterized by the intensity distribution in the plane of MODAN (*x,y,0*)

$$I_0(x,y) = \exp\left[-\frac{2\left(x^2+y^2\right)}{\sigma_{00}{}^2}\right] \qquad (15)$$

and by a phase distribution assumed to be constant, which is a good approximation in the vicinity of the beam waist. As an invariant multimode beam under investigation the sum of two Gauss-Hermite modes with numbers (4,0) and (2,2) was chosen:

$$\chi_4(x,y) = \psi_{40}(x,y) + \psi_{22}(x,y) \qquad (16)$$

So, the complex transmission function of the manufactured DOE can be written as:

$$T(x,y) = \frac{\chi_4(x,y)}{\sqrt{I_0(x,y)}} \qquad (17)$$

Fig. 2 presents the theoretical amplitude distribution $|\chi_4(x,y)|$ in the cross-section of the beam to be formed. Fig. 3 presents the theoretical phase distribution $\arg(\chi_4(x,y))$ in the cross-section of the beam to be formed. Fig. 4 presents a result of computer simulation for the amplitude distribution $|\chi_4(x,y)|$ in the cross-section of the beam after propagation through an additional Fourier-stage. This simulation was realized by means of the software "QUICK-DOE" developed at the Image Processing Systems Institute of the Russian Academy of Sciences [9]. The amplitude structure conservation could be confirmed.

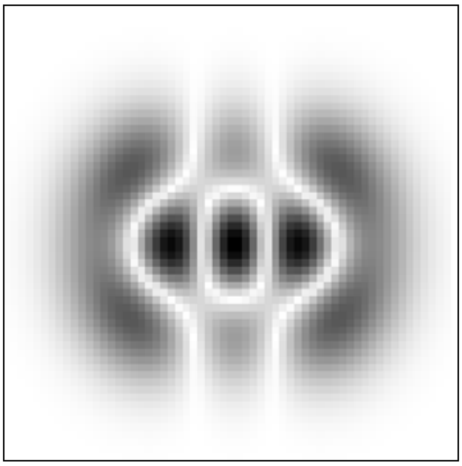

Fig. 2: Theoretical amplitude distribution in the cross-section of the invariant beam

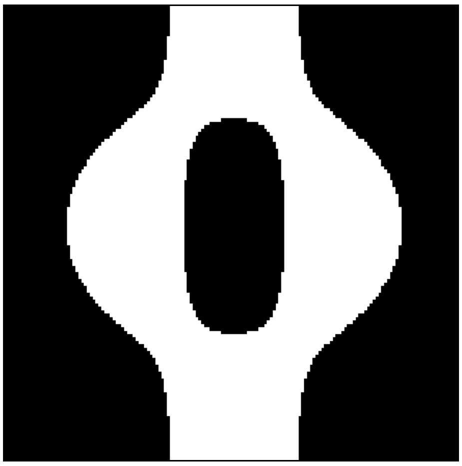

Fig. 3: Theoretical phase distribution in the cross-section of the invariant beam (black corresponding to phase 0, white - to π*)*

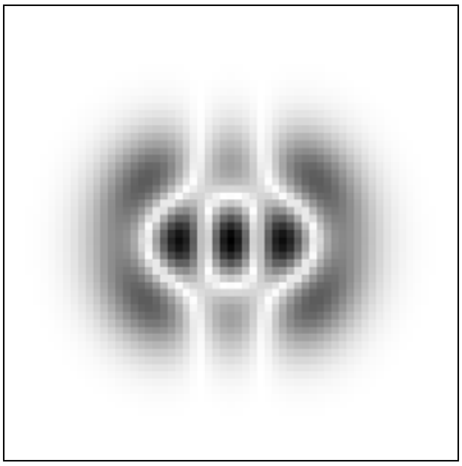

Fig. 4: Amplitude distribution in the cross-section of the invariant beam after propagation through a Fourier-stage (computer simulation)

The element was coded as a grating with 33.3 lines/mm, with a rectangular shaped carrier function "slowly" modulated across the aperture (generalized Kirk-Jones method, [1]). The grating was calculated with a resolution of 1024*1024 pixels, with a pixel size of 3 microns.

The selected element was designed to work with an external Fourier lens, and for the calculation the following parameters had been used: a wavelength of the illuminating beam of λ = 632.8 nm, a Gaussian input beam radius of σ_{00} = 0.525 mm, a fundamental radius of σ = 0.5 mm for the generated invariant multimode beam in the focal plane of the Fourier lens, a focal length of this lens of f = 452 mm. The distance between the used zero-order spot and the corresponding plus/minus first order (parasitical) spots should be 9.52 mm. During calculation as criterion for reconstruction quality the energy efficiency was applied:

$$\eta = \frac{\iint\limits_G \left|\chi_4(x,y)\right|^2 dxdy}{\iint\limits_D I_0(u,v)dudv}, \tag{18}$$

where $\chi_4(x,y)$ is the complex amplitude distribution generated by the calculated element and D is the MODAN's aperture domain. The calculation resulted in a value of η = 0.18 for energy efficiency. The essential result of the calculation was the phase distribution itself representing the MODAN to be manufactured. Fig. 5 depicts a half-tone mask of the element calculated for the set of parameters listed above.

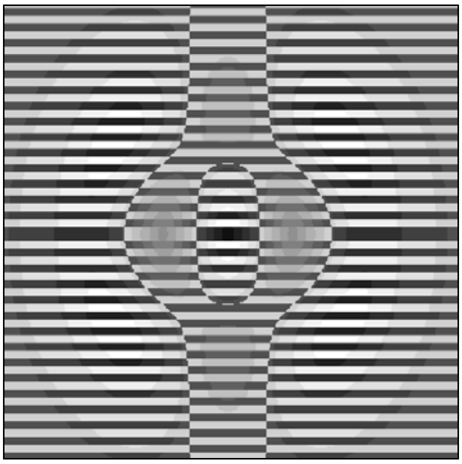

Fig. 5: Phase mask of manufactured MODAN (central part)

The calculated element has been manufactured as a multilevel-binary surface profile by (variable dose) electron-beam direct-writing into a PMMA resist film and a subsequent development procedure of the resist. The final element consists of a fused silica substrate coated with the structured PMMA film. The continuous phase profile had to be transferred into a corresponding surface profile, which in turn had to be approximated by a step-like structure. For the element under discussion, we used a 15 step/16 level approximation of the continuous profile. In connection with the specific technological approach we used, every single step in depth of the staircase profile had to be generated by an individual „etching“ process. Therefore, 15 isolated binary masks had to be generated (as data fields only - not physically!) by software, starting from the continuous surface profile. The 15 dose levels each corresponding to one of the final surface levels were realized by 15 times application of a binary electron beam writing process, using a commercial ZBA 23 system. After a proper development procedure applied to the PMMA film, the designed staircase surface profile turned up and could be controlled by means of an optical profiler MICROMAP 512. The manufactured MODAN had to demonstrate its ability to realize the invariant multimode forming it was designed for in a series of optical experiments. The set-up schematically shown in Fig. 6 allowed to measure the intensity distribution generated by the element in the focal plane of the Fourier lens. A typical result of this investigation is depicted in Fig. 7.

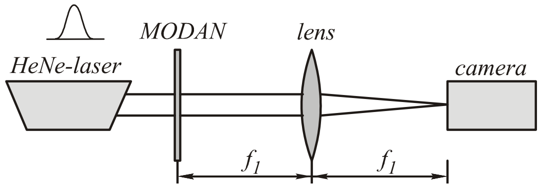

Fig. 6: Set-up for MODAN characterization by measurement of the formed intensity distribution

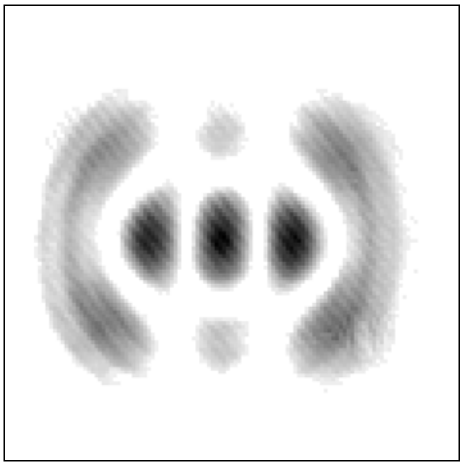

Fig. 7: Amplitude as result of measurement in the MODAN's focal plane

Comparison with theoretical results (Figures 2 and 4) manifests on one hand a very good correspondence, for example regarding the fundamental radius of the generated invariant multimode beam, and regarding the overall shape. On the other hand in experiment a certain asymmetry of the intensity distribution is visible, which is not predicted by theory for the ideal element. The set-up in Fig.6 was used furthermore for estimating the energy efficiency regarding Eq. (18): For this purpose, the CCD-camera had to be replaced by a power meter, and parasitical diffraction orders in the focal plane of Fourier lens had to be camouflaged properly by a stop with an adjustable circular aperture. The energy efficiency in this experiment was found to be η = 0.16, which is somewhat less than the result η = 0.18 as obtained by simulation. This difference may be accounted for by technological errors during MODAN fabrication as well as by quantization and discretization errors. To demonstrate the "invariant" character of the complex amplitude distribution generated by the MODAN, further evidences were needed: one possibility was to submit this distribution to a further Fourier transformation. A complex amplitude distribution representing any invariant multimode beam should have retained its spatial structure during this procedure, while changing its fundamental radius according to Eq. (12). The optical set-up used for this experiment is schematically shown in Fig. 8. A typical result representing a measured amplitude distribution in the focal plane of second Fourier lens is given in Fig. 9. An additional evidence for mode-like behavior could be received by investigating the generated phase distribution: For this purpose we used an interferential set-up, where the Gaussian (0,0) beam was applied as reference beam. An interferential fringe pattern could be generated by introducing a slight tilt between the two interferometer arms. Then - caused by the phase jump of π appearing between the different parts of the invariant beam - a corresponding shift between different parts of fringe pattern system should occur. Additionally to the estimated shift between fringe systems in different areas of beam cross-section we can see a different tilt of the fringe systems on left and right side, respectively, of this cross-section. This effect is probably caused by manufacturing imperfections, and has to be investigated further in more details.

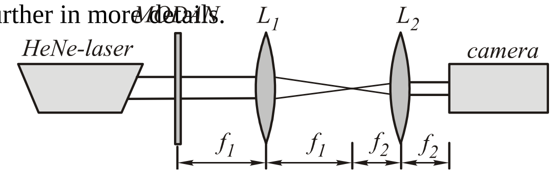

Fig. 8: Scheme with invariant beam passing through an additional Fourier-stage

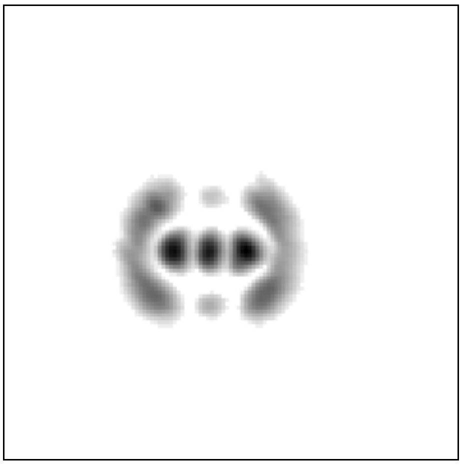

Fig. 9: Amplitude distribution measured in the output plane of the Fourier-stage

A typical measured result of such a fringe pattern is shown in Fig. 10. Furthermore we applied a digital method following [10] to calculate the phase distribution in the focal plane of first Fourier lens: the phase distribution has been restored from measured intensity distribution in the input plane and in the output plane, respectively, of second Fourier lens, by 26 iterations following the procedure of [10]. The set-up used for registration of the pairs of intensity distributions is similar to that of Fig. 6 and Fig. 8. After finishing the last of the 26 iterations, the integral root-mean-square deviation between measured and calculated amplitude distributions achieved less then 16%. The calculated ("restored") phase distribution in the input and output planes of second Fourier lens is presented in Fig. 11.

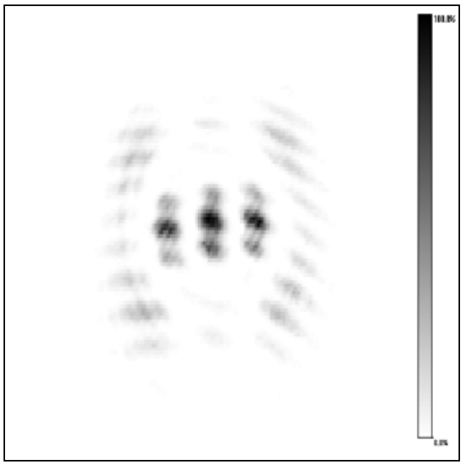

Fig. 10: Intensity distribution as result of interferential investigation

a)

b)

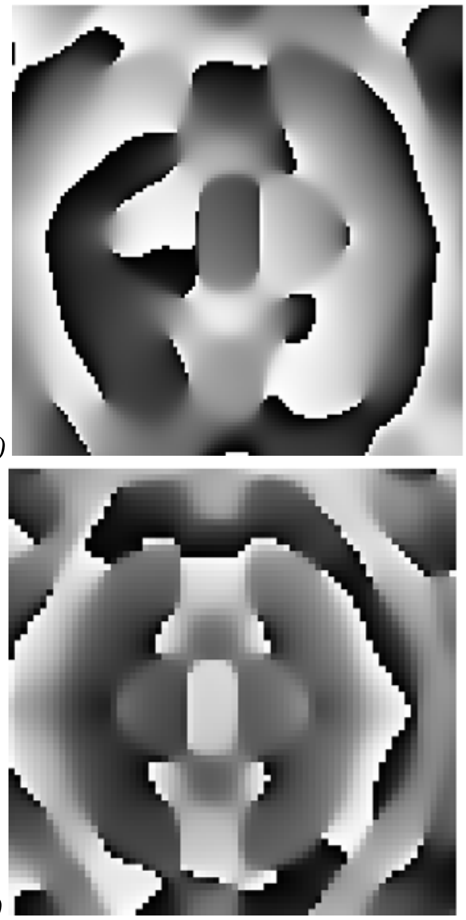

Fig. 11: Results of the invariant beam phase iterative restoration a) restored phase distribution in the input plane of the second Fourier-stage, b) restored phase distribution in the output plane of the second Fourier-stage

So, the invariant beam amplitude structure conservation feature was investigated by intensity distribution measurements in the input and output planes of the Fourier-stage. Invariant beam intensity investigation results showed good stability of the invariant beam intensity structure during beam propagation. The phase structure of formed invariant beam was investigated by both digital and interferometric methods. The iterative phase restoration results are in good agreement with the experimental phase investigation results. Invariant beam phase investigation results showed stability of the invariant beam phase structure during beam propagation through Fourier-stage.

Besides, in this work the comparison of invariant and "non-invariant" beams behavior was implemented by methods of calculation and optical experiments too. The multimode beams consisting of more than one modes having different propagation constant values were called "non-invariant beams". It is very interesting to compare invariant beams with "non-invariant" ones. For this purpose, a MODAN generating a mode combination of Gauss-Hermite modes with numbers (0,1) and (2,2) with equal weights was designed. All physical parameters were chosen as for the MODAN described before. Corresponding results of optical and computational investigations are presented in Fig. 12 – Fig. 15.

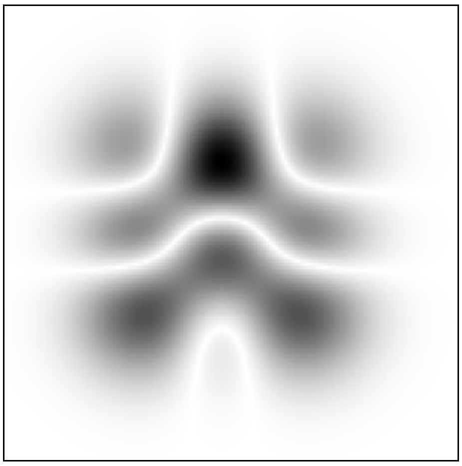

Fig. 12: Theoretical amplitude distribution in the cross-section of the "non-invariant" beam (Gauss-Hermite (0,1)+(2,2))

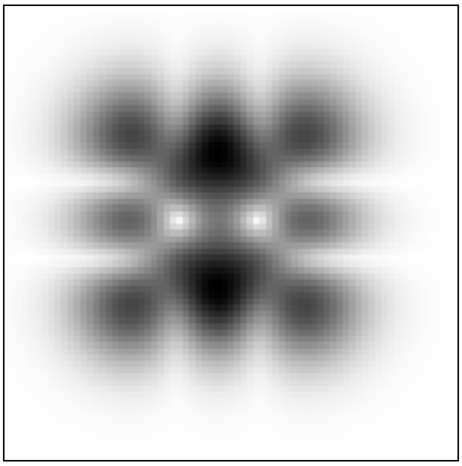

Fig. 13: Theoretical amplitude distribution in the cross-section of the "noninvariant beam" after propagation through a second Fourier-stage (computer simulation)

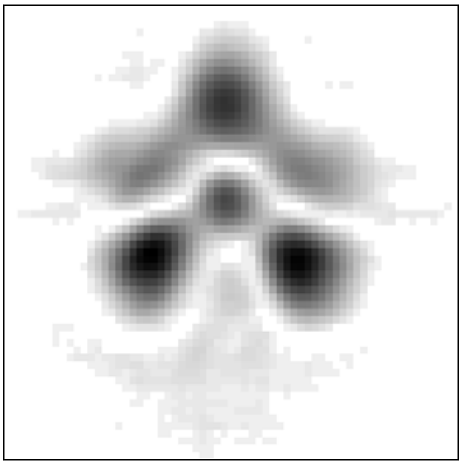

Fig. 14: Amplitude as result of measurement in the focal plane of MODAN

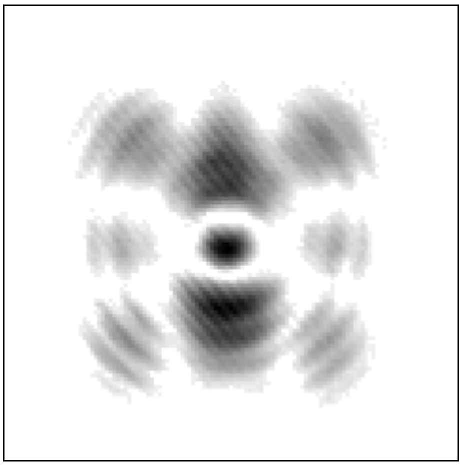

Fig. 15: Amplitude as result of measurement in the output plane of second Fourier-stage

4. Laser modal beam selection by DOE in telecommunication tasks

Modern communication systems are characterized by users' demand for a number of channels as large as possible. If we think about laser light as carrier for the information, the application of modes turns out to be a key to this problem. Communication systems so far are based on the application of only different longitudinal modes as independent channels [11,12], exploiting wavelength as distinctive feature. In other words, wavelength-selective optical filters serve both as channel generators and multiplexers. The real-time one-fiber image transfer system described in [13] is using white light decomposition into spectral components. The component selection in [13] is based on application of segmented DOE. Each of DOE segments in [13] is matched to the corresponding spectral range. Recent achievements in the development of dielectric band pass filters with bandwidth $\Delta\lambda \le 1nm$ as mass products remarkable enlarged the number of available channels in such systems. Our suggestion is now to employ different **transversal** modes as non-interacting transmission channels, instead of (or may be later - additionally to) the common longitudinal modes. This suggestion is motivated by the fact that certain novel Diffractive Optical Elements (DOEs), named as MODANs, turned out to be able to generate desired transversal modes respectively mode mixtures, to transform given modes into other modes, to separate different modes from each other, or to analyze mode mixtures of unknown composition regarding the mode content [1]. The theory of these MODANs is described in [1] in detail. Even though only a small portion of all the elements treated there has been manufactured and experimentally investigated till now, and though many of them suffer from their low diffraction efficiency, caused by applied coding methods, such MODANs represent very helpful tools to deal with transversal modes. Recent promising results [2,5] regarding calculation, manufacture and experimental investigation of effective mode-transforming MODANs, motivated us to realize and to investigate a complex system representing a model of a multichannel communication device based on application of **transversal** modes [3]. Note that use of invariant beams results in essential improvement of fiber channel capacity since the data transfer occurs in a parallel fashion through several invariant beams, with no pulse blurring observed in each branch. In this way one can obtain modal channel multiplexing together with common-used frequency multiplexing. It is interesting to note that "spectral" DOEs [14,15] which can operate with some wavelengths in parallel can be used for such purpose in future.

At the same time, the choice of invariant beams as a base for construction of multichannel telecommunication system allows to use as free parameters for calculation of the phase DOE generating the set of several invariant beams in parallel not only the phase shifts between different invariant multimode beams, but additionally internal shifts and energy redistribution between separate modes within invariant multimode beam as well [5].

5. Design of a multichannel waveguide telecommunication system with high energy efficiency

Let us suppose that we have to construct a system consisting of N_k independent digital information channels (Fig. 16) transferred through an ideal lens-like medium, where different channels should be represented by different invariant multimode Gauss-Hermite beams according to Eq. (14). Let us assume further a "homogeneous" energy distribution between the N_k channels $B_1 = B_2 = ... = B_{N_k}$, with

$$B_i = \sum_{(p,l)} \left|\tilde{C}^{i}_{pl}\right|^2 ,$$

and

$$\sum_{i=1}^{N_k} B_i = E_0 \qquad (19)$$

where $\tilde{C}^{i}_{pl}$ - are their mode coefficients of corresponding i th invariant multimode beam described by Eq. (14), and E_0 is the energy of the collimated laser source **L**.

We will not take into account energy losses connected with absorption and Fresnel reflection. The general number of invariant beams which can be used is the *cut-off number* $(p+l)_{\max} = N_{cut}$ of the waveguide F [4]. For spatial separation and subsequent time modulation we realize the following decomposition which is a modification of one proposed in [16] before:

$$A(x,y)\exp(i\varphi(x,y)) = \sum_{j=1}^{N_k} exp(i2\pi\nu_j x) \sum_{p+l=j} \tilde{C}^{j}_{pl}\psi^{j}_{pl}(x,y)$$

$$N_k \le N_{cut}, \qquad (20)$$

where $A(x,y)$ is the amplitude distribution in the cross-section of the illuminating collimated beam, $\varphi(x,y)$ is the phase function of the MODAN M and ν_j is the carrier frequency introduced for spatial beam separation.

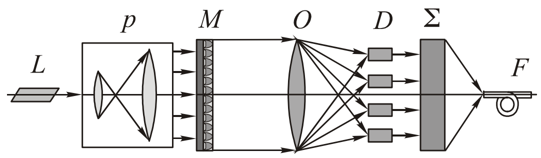

Fig. 16: General scheme of a multichannel waveguide telecommunication system. L - laser light source, P - collimator, M - MODAN, O - Fourier stage, D - set of modulators, F - ideal parabolic index fiber.

To find the coefficients $\tilde{C}^{j}_{pl}$ in Eq. (20), we can use any recursive optimization procedure (for example, stochastic procedure [17]) minimizing the functional:

$$\delta_m = \sum_{j}^{N_k} \left| \frac{1}{N_k} - \sum_{p+l=j} \left|\tilde{C}^{jm}_{pl}\right|^2 \right| \qquad (21)$$

with the result of coefficient estimation after m th recursive iteration procedure:

$$\tilde{C}^{jm}_{pl} = \iint_G A(x,y)\exp(i\varphi_m(x,y)) \bullet$$

$$\bullet\, \psi^{j}_{pl}(x,y)\exp(i2\pi\nu_j x)dxdy, \qquad (22)$$

where $\varphi_m(x,y)$ is the DOE's phase distribution after *m-th* iteration.

6. Limits of the scalar theory applicability to description of gradient-index waveguides

The diffraction calculation is based on solving (in different ways) a wave equation adequately describing the light propagation (in particular, diffraction) only in one of the polarization states: TE- or TM- polarization. This is due to the fact that one can fully define the electromagnetic wave knowing at least its two longitudinal components - the electrical (TE-wave) and the magnetic (TM-wave) one [7]. Setting up and solving the wave equation for one longitudinal component is substantiated for homogeneous index medium or a medium with cylindrical inhomogeneities. Otherwise, the emergence of differently polarized modes becomes inevitable. In this case, the solution of the wave equation yields a mispresented modal composition which will involve not really present higher-order modes and lack real, but differently polarized modes. Hence, we infer that diffraction theory becomes inapplicable for calculation of light propagation through a waveguide with non-cylindrical inhomogeneities having a geometrical size greater than quarter of the wavelength. There is a more detailed analysis of diffraction theory applicability for gradient waveguide analysis in [4].

7. Conclusions

In this paper the principal possibility of invariant beam forming by diffractive optical elements was shown. Fundamental properties of invariant multimode beam were investigated by methods of calculation and optical experiment. There is a good agreement between theory and experimental results. The invariant beam amplitude-phase structure conservation feature was investigated by intensity distribution measurements in input and output planes of an additional Fourier-stage. Invariant beam intensity investigation results showed the stability of invariant beam intensity structure during beam propagation. Phase structure of formed invariant beam was investigated by both digital and interferometric methods. The iterative phase restoration results are in good agreement with interferometric phase investigation results. Invariant beam phase investigation results showed stability of the invariant beam phase structure during beam propagation. The comparison of invariant and "non-invariant" beams behavior was made by methods of calculation and optical experiments too. Invariant beams fundamental properties were investigated (self-reproduction, amplitude-phase structure stability) promising good prospectives for application of invariant beams in future high efficient telecommunication systems.

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9. References

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