EXPERIMENTAL INVESTIGATION OF MULTIMODE DISPERSIONLESS BEAMS

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Abstract

Laser light modes are beams in whose cross-section the complex amplitude is described by *eigenfunctions* of the operator of light propagation in the waveguide medium. The fundamental properties of modes are their orthogonality and their ability to retain their structure during propagation for example in a *lenslike medium* or in *free space*. Developed diffractive optical elements (DOEs) of *MODAN*-type open up new promising potentialities of solving the tasks of generation, transformation, superposition of different laser modes and their combinations. Now we present new results obtained by synthesis and investigation of beams consisting of more than one two-dimensional Gaussian laser modes with the same value of propagation constant - *multimode dispersionless beams*.

Introduction

Laser light modes are beams in whose cross-section the complex amplitude is described by *eigenfunctions* of the operator of light propagation in the waveguide medium.

The fundamental property of modes are the property of retaining their structure and orthogonality during the propagation in a waveguide medium (for example *free space* or *lens-like medium*). Developed diffractive optical elements (DOEs) of *MODAN*-type [1,2] open up new promising possibilities of solving the tasks of generation, transformation, superposition of different modes and their combinations. In [3] we presented a MODAN, capable to transform a Gaussian (0,0) input beam into a combination of two Gauss-Hermite modes having the same value of propagation constant taking with fixed zerovalued intermode phase shift. Now we present new results, obtained by synthesis and investigation of beams, consisting of two modes with the same value of propagation constant (*multimode dispersionless beams*) with changing (from time to time) of intermodal phase shift. Multimode dispersionless beams, excited by phase DOEs with high efficiency, can be used in the future for optical communication purposes because of the absence of pulse enlargement phenomena [4] caused by modal dispersion. Use of the phase intermodal shifts as additional degree of freedom allows the possibility of design of the DOE with high energy efficiency.

We present experimental investigations of multimodal dispersionless beam propagation through Fourier stage. The results demonstrate promising perspectives for the selected concept in future.

1. Basic formalism

Modal beams do not change their spatial structure in a proper waveguide medium. Every mode gets its own phase delay, proportional to the optical path length and to the *propagation constant*. Thus, in the case of a graded-index optical fiber, the phase delay is continuously accumulated during the mode propagation. Furthermore, in this case guided modes reproduce their modal configuration after each path of sufficient length in the fiber. Let us set the Cartesian coordinates $(x, y, z) = (\mathbf{x}, z)$ in the medium of beam propagation. The two-dimensional vector $\mathbf{x} = (x, y)$ represents the transverse coordinates; z is the longitudinal coordinate along the optical axis. Guided modes under consideration are thought as located within the domain $\mathbf{x} \in G$ in the beam cross-section. We use the scalar representation of light field and the scalar diffraction theory without any consideration of polarization effects. Thus, we describe the monochromatic or quasi-monochromatic field by the complex amplitude $w(\mathbf{x}, z)$ with wavelength λ or wavenumber $k = \frac{2\pi}{\lambda}$. Besides, by default we will suggest that waveguide medium (gradient index waveguide or free space) under consideration has a *invariant translation feature,* e.g. its characteristics do not change along the z - axis. Let's consider the Helmholtz equation for gradient index medium description

$$\nabla_\perp^2 w(x,y,z) + \frac{\partial^2 w(x,y,z)}{\partial^2 z} + \\ + n^2(x,y)\cdot k^2 \cdot w(x,y,z) = 0 \qquad (1)$$

with the initial values $w\big|_{z=0} = w(x, y, 0)$ of the complex amplitude, where $\nabla_\perp = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$ is the transverse differential Hamiltonian operator. If finite diameter of waveguide is taken into account, then certain additional boundary conditions appear at the interface between core and cladding. According to [1], the modes of a graded-index optical fiber (Fig. 1) have a plane wavefront and obey equation

$$\nabla_{\perp}^{2}\psi_{pl}(x,y)+\left[k^{2}n^{2}(x,y)-\beta_{pl}^{2}\right]\cdot\psi_{pl}(x,y)=0\,. \quad (2)$$

For any given distance z we have

$$w(x,y,z)=\gamma_{pl}\cdot\psi_{pl}(x,y), \quad (3)$$

$$\gamma_{pl}=\exp\left(i\left[\beta_{pl}+i\alpha_{pl}\right]z\right), \quad (4)$$

where β_{pl} is the propagation constant and α_{pl} is the coefficient of attenuation for the mode ψ_{pl}. Thus, the modes of graded-index optical fiber satisfy the eigenvalue equation (2) for any distance z passed by light. Eigenvalues are specified by Eq. (4). It must be noted, that Eq. (3) describes the modal self-reproduction, that occurs with a constant scale regarding the Cartesian coordinates (x,y). For the propagation in a fiber with quadratic refractive index described in the form

$$n^{2}(r)=n_{1}^{2}\left(1-2\Delta\frac{r^{2}}{a^{2}}\right), \quad (5)$$

where n_1 – refractive index at the axe of the core, a – radius of the core, we have

$$\gamma_{pl}(z)=\exp(i\beta_{pl}z)\,. \quad (6)$$

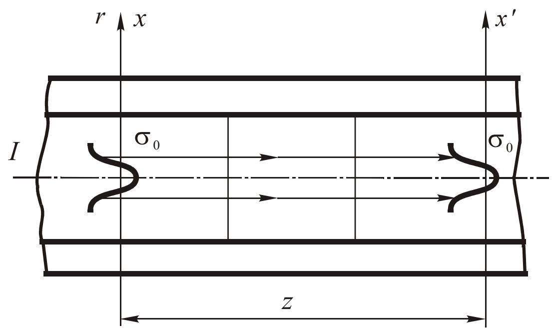


Fig. 1. Modes in a graded-index optical fiber with parabolic profile

Gauss-Hermite modes are eigenfunctions of the propagation operator in a waveguide medium with parabolic profile according to Eq. (5). As solutions of the Helmholtz equation for a waveguide with parabolic profile in the Cartesian coordinate system we obtain the well-known formulae for the complex amplitude of Gauss-Hermite "GH" modes described in [4],

$$\psi_{pl}(x,y)=E_{pl}\cdot H_{p}\left(\frac{\sqrt{2}x}{\sigma}\right)\cdot H_{l}\left(\frac{\sqrt{2}y}{\sigma}\right)\cdot\exp\left[-\frac{x^{2}+y^{2}}{\sigma^{2}}\right], \quad (7)$$

where $H_p(.)$ - is the Hermite polynom of p th order, σ is the mode fundamental radius and

$$E_{pl}=\frac{1}{\sigma}\cdot\sqrt{\frac{2}{\pi\cdot 2^{p+l}\cdot p!\,l!}} \quad (8)$$

is a normalization constant. The propagation constant β_{pl} for Gaussian modes [1] is

$$\beta_{pl}=\sqrt{k^{2}n_{1}^{2}-\frac{4}{\sigma^{2}}(m+1)}\,, \quad (9)$$

where the two-dimensional integral mode number follows $m=p+l$. Let us remember the well-known mode fundamental features [1,4]. Being natural (normal) or eigen-oscillations, the modes of a waveguide may be characterized by the following invariant and optimal properties:

1. *Each waveguide medium can be characterized by a* *discrete* *set* $\{\psi_{pl}(x,y)\}$ *of its eigen-oscillations - modes.*
2. *Modes are the unique two-dimensional base-functions that conserve the orthogonality during guided propagation in their native waveguide media.*
3. *Modes are the unique two-dimensional base-functions that conserve amplitude-phase structure during guided propagation in their native waveguide media.*

Note, that for two-dimensional waveguide cross-section the same particular value of propagation constant β_{pl} can correspond to more than one different modal functions: for instance, it is easy to see that two Gauss-Hermite modes with numbers (p,l) and (l,p) will have the same value $m=p+l=l+p$. In general, by virtue of Eqs. (3-6) and Eq. (9), under condition of $a_{pl}=0$ (no attenuation) the propagation of any linear combination $\chi_m(x,y)$ of more than one different Gaussian modes $\psi_{pl}(x,y)$ each with the same value of propagation constant β_{pl}

$$\chi_{m}(x,y)=\sum_{p+l=m}\tilde{C}_{pl}\psi_{pl}(x,y)=\sum_{p=0}^{m}\tilde{C}_{p(m-p)}\psi_{p(m-p)}(x,y)=\sum_{p=0}^{m}\left|\tilde{C}_{p(m-p)}\right|\exp\left(i\arg\left[\tilde{C}_{p(m-p)}\right]\right)\psi_{p(m-p)}(x,y) \quad (10)$$

would be similar to propagation of isolated mode. So, a beam with a cross section corresponding to Eq. (10) will have no change in its amplitude-phase structure during propagation in waveguide medium.

Laser beams having cross-section which can be described by Eq. (10) have been called *multimode dispersionless beams (or invariant multimode beams* [3]*)*. Lets us note that each of coefficients $\tilde{C}_{p(m-p)}$ in general case can be zerovalued. So, multimode dispersionless beams may be characterized by the following invariance properties [3]:

1. *Each* *discrete* *waveguide mode set* $\{\psi_{pl}(x,y)\}$ *is able to generate a* *continuous* *set of multimode dispersionless beams because of the continuous variety of complex-valued coefficients* $\tilde{C}_{pl}$ *in Eq.* (10).
2. *Self-reproduction: multimode dispersionless beam does not change its amplitude-phase structure and size during propagation in a proper waveguide medium.*

3. *Gaussian multimode dispersionless beam does not change its amplitude-phase structure during propagation in free-space.*
4. *Gaussian multimode dispersionless beam does not change its amplitude-phase structure during propagation through a Fourier-stage, whereas the fundamental radius or self-mode parameter changes [1].*
5. *A multimode dispersionless beam can propagate through a waveguide without pulse enlargement effect [4].*

2. Multimode dispersionless beam investigation by methods of optical experiments

In [3] we presented a modan, capable to transform a Gaussian (0,0) input beam into an combination of two Gauss-Hermite modes having same value of propagation constant taking with fixed intermodal phase shift. Now we present new results, obtained by synthesis and investigation of beams, consisting of two modes with the same value of propagation constant (*multimode dispersionless beams*) with possibility to change the intermodal phase shift during experiment. In order to demonstrate fundamental properties of multimode dispersionless beams, we designed a modan which should be able to transform one single transversal mode into the two different modes having same propagation constant in different diffractive orders. For the input beam we selected the Gauss (0,0) mode of He-Ne laser (wavelength is λ=0.63 μm) characterized by the intensity distribution in the plane of MODAN

$$I_0(x,y) = \exp\left[-\frac{2\left(x^2+y^2\right)}{\sigma_{00}{}^2}\right] \qquad (11)$$

and by a phase distribution assumed to be constant, which is a good approximation in the vicinity of the beam waist. Modan was calculated to be able to form modes Gauss-Hermite (2,2) and Gauss-Hermite (4,0) (m=4) in different diffractive orders in the output planes. The complex transmission function of modan *T(x,y)* can be written as follows

$$T(x,y) = \frac{1}{\sqrt{I_0(x,y)}}\left(\psi_{22}(x,y)\exp(i2\pi\nu_{22}x) + \psi_{40}(x,y)\exp(i2\pi\nu_{40}x)\right), \qquad (12)$$

where ν_{22}, ν_{40} - carriers introduced for spatial separation of unimodal beams. The well-known Kirk-Jones [5] method was used for coding of complex transmission function *T(x,y)* of modan into a pure phase function of modan $\phi(x,y)$.

The calculated element has been manufactured as a multilevel surface profile by (variable dose) electron-beam direct-writing into a PMMA resist film and a subsequent development procedure of the resist. The final element consists of a fused silica substrate coated with the structured PMMA film. The continuous phase profile had to be transferred into a corresponding surface profile, which in turn had to be approximated by a step-like structure. For the element under discussion, we used a 15 step/16 level approximation of the continuous profile. The 15 dose levels each corresponding to one of the final surface levels were realized by 15 times application of a binary electron beam writing process, using a commercial ZBA 23 system (Carl Zeiss, Jena, Germany).

The set-up schematically shown in Fig. 2 allowed to measure the intensity distribution in the cross-section of beam combined from two different modes taking with different phase shifts. To demonstrate the "invariant" character of the complex amplitude distribution, further evidences were needed: one possibility was to submit this distribution to a further Fourier transformation (lens L2 was used). A complex amplitude distribution representing any multimode dispersionless beam should have retained its spatial structure during this procedure, while changing its fundamental radius. Camera 1 and Camera 2 have been synchronized by computer. Positions of Camera 1 and Camera 2 corresponding exactly to both focal planes of the lens L2.

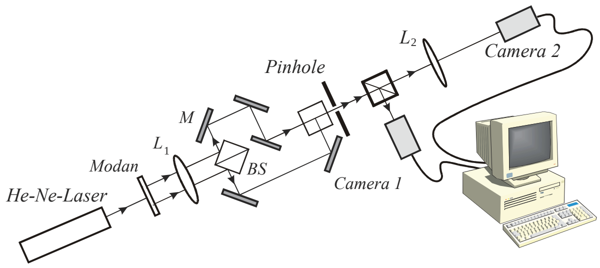
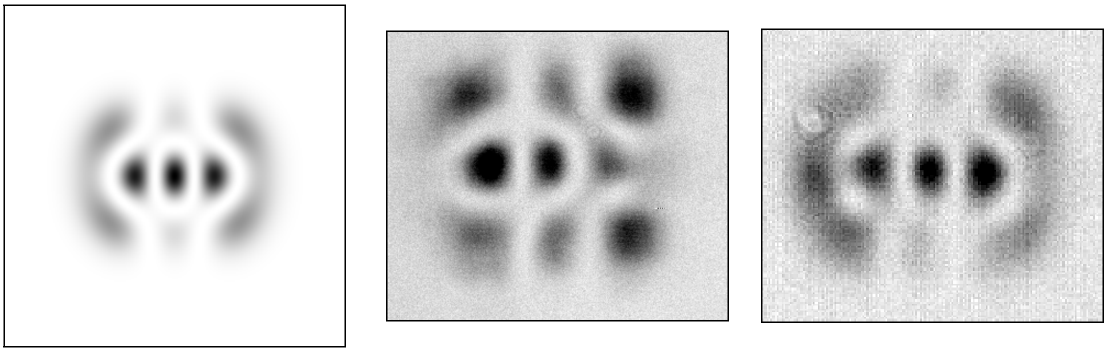


Fig. 2. Set-up for experimental investigation of multimode dispersionless beams. BS – beamsplitter, L_1, L_2 –lenses, M – mirrors, Camera 1, Camera 2 – CCD-cameras synchronized by computer.

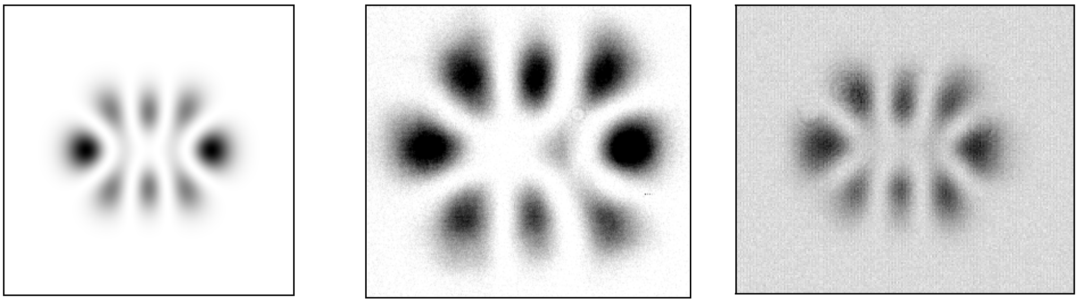
Pixel size of the both applied cameras was 11.0 μm x 11.0 μm. System of mirros M has been used for fine control of intermode phase shift. The focal distance of the lens L2 was f=300 mm. The fundamental radius of modes in the plane of Camera 1 has been measured as σ_1=0.61 mm which is in the good agreement with theoretical estimation σ_1=0.62 mm. The fundamental radius of modes in the plane of Camera 2 has been measured as σ_2=0.33 mm.

A typical results of this investigation are depicted in Fig.3 (corresponding to $\Delta\varphi = \arg\left[\tilde{C}_{22}\right] - \arg\left[\tilde{C}_{40}\right] \approx 0$) and Fig. 4 (corresponding to $\Delta\varphi = \arg\left[\tilde{C}_{22}\right] - \arg\left[\tilde{C}_{40}\right] \approx \pi$).



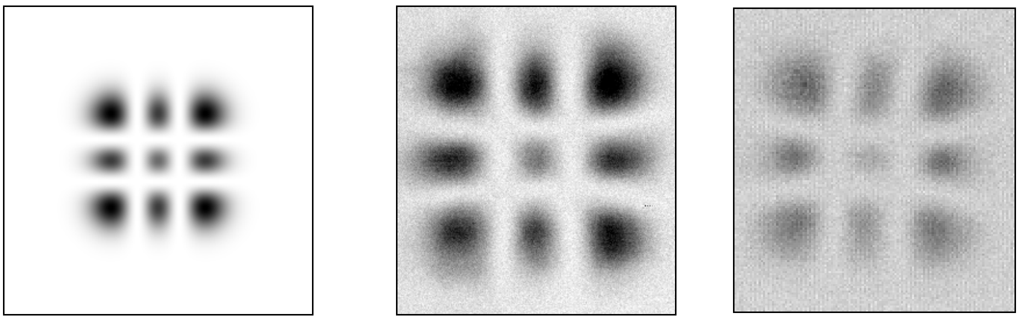
a *b* *c*

Fig. 3. Intensity distribution in the cross-section of two-modal beam (Gauss-Hermite (4,0)+(2,2)) with intermodal phase shift value Δφ=0: a) result of computer simulation, b),c) – intensity distributions measured in different focal planes of the lens L2



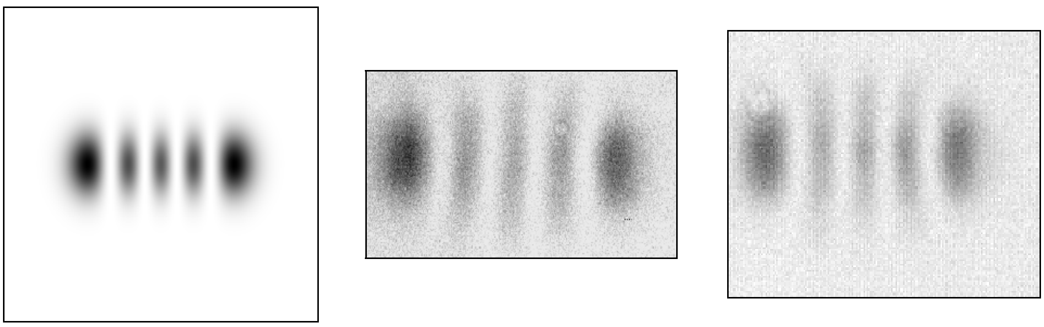
a *b* *c*

Fig. 4. Intensity distribution in the cross-section of two-modal beam (Gauss-Hermite (4,0)+(2,2)) with intermodal phase shift value Δφ=π: a) result of computer simulation, b), c) – intensity distributions measured in different focal planes of the lens L2.



a *b* *c*

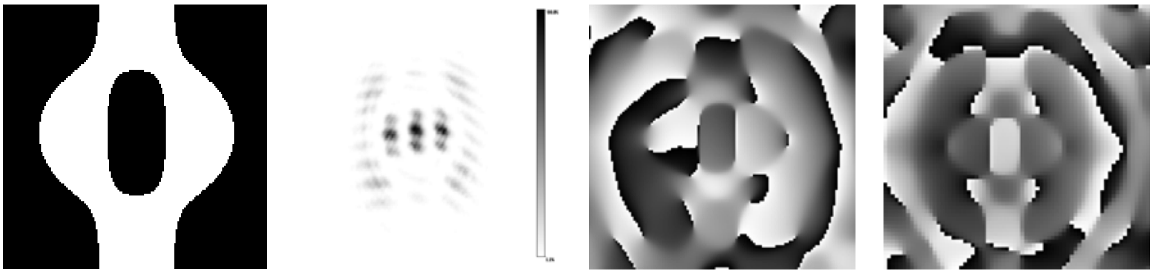
Fig. 5. Intensity distribution in the cross-section of unimodal beam (Gauss-Hermite (2,2)): a)result of computer simulation, b),c) – intensity distributions measured in different focal planes of the lens L2.



a *b* *c*

Fig. 6. Intensity distribution in the cross-section of unimodal beam (Gauss-Hermite (4,0)): a)result of computer simulation, b),c) – intensity distributions measured in different focal planes of the lens L2.

All results of measurement here and further are given without taking account the scale of picture.The blocking of one of the two unimodal beams leads to results depicted in Fig. 5 and Fig. 6. So, the multimode dispersionless beam amplitude structure conservation feature has been investigated by intensity distribution measurements in the input and output planes of the Fourier-stage. Intensity investigation results showed good stability of the multimode dispersionless beam intensity structure during beam propagation. In before in [3] we applied an interferometrical methods as well as digital procedure following [6] to estimate the phase distribution in different cross sections of multimode dispersionless beam. Typical results are depicted in Fig. 7. So, multimode dispersionless beams phase investigation showed good stability of multimode dispersionless beam phase structure during beam propagation through Fourier-stage too.



a *b* *c* *d*

Fig. 7. Phase distribution in the cross-section of two-modal beam (Gaussian-Hermite (4,0)+(2,2)) with intermodal phase shift value Δφ=0: a) as a result of computer simulation (black corresponding to phase 0, white - to π), b) as a result of interferometrical investigation, c),d)- phase distributions iteratively restored in different planes of Fourier-stage.

Multimode dispersionless beams fundamental properties were investigated (self-reproduction, amplitude-phase structure stability) promising good prospectives for application of such beams in future high efficient telecommunication systems.

3. Design of a multichannel waveguide telecommunication system with high energy efficiency

Let us suppose that we have to construct a system consisting of N_k independent digital information channels (Fig. 8) transferred through an ideal lens-like medium without *mode dispersion* with *energy efficiency* as high as possible.

Lets us try to use different multimode dispersionless Gauss-Hermite beams for representation of different channels.

Let us assume also a "homogeneous" energy distribution between the N_k channels $B_0 = B_1 = ... = B_{N_k -1}$, with

$$B_i = \sum_{p=0}^{i} \left| \tilde{C}_{p(i-p)} \right|^2 ,$$

and

$$\sum_{i=0}^{N_k -1} B_i = E_0 \qquad (13)$$

where $\tilde{C}_{p(i-p)}$ - are mode coefficients of corresponding *i*-th multimode dispersionless beam described by Eq. (10), and E_0 is the energy of the collimated laser source *L*.

We will not take into account energy losses connected with absorption and Fresnel reflection. The general number of invariant beams which can be used is the *cut-off number* $(p+l)_{\max} = N_{cut}$ of the waveguide F.

For spatial separation and subsequent time modulation we realize the following decomposition which is a modification of one proposed in [7] before:

$$A(x,y)\exp(i\phi(x,y)) = \\ = \sum_{j=0}^{N_k -1} \exp(i2\pi\nu_j x) \sum_{p=0}^{j} \tilde{C}_{p(j-p)} \psi_{p(j-p)}(x,y)$$

$$N_k \le N_{cut} , \qquad (14)$$

where $A(x,y)$ is the amplitude distribution in the cross-section of the illuminating collimated beam, $\phi(x,y)$ is the phase function of the MODAN *M* and ν_j is the carrier frequency introduced for spatial beam separation.

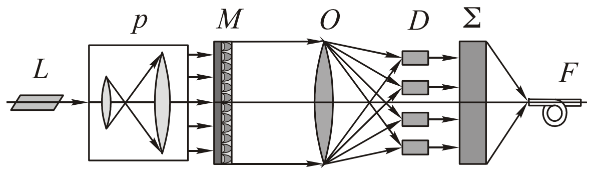


Fig. 8. General scheme of a multichannel waveguide telecommunication system: L - laser light source, P - collimator, M - MODAN, O - Fourier stage, D - set of modulators, F - ideal parabolic index fiber.

To find the coefficients $\tilde{C}_{p(j-p)}$ in Eq. (14), we can use any recursive optimization procedure minimizing the functional:

$$\delta_m = \sum_{j=0}^{N_k -1} \left| \frac{1}{N_k} - \sum_{p=0}^{j} \left| \tilde{C}_{p(j-p)} \right|^2 \right| \qquad (15)$$

with the result of coefficient estimation after *m* th recursive iteration procedure:

$$\tilde{C}_{p(j-p)} = \iint_G A(x,y)\exp(i\phi_m(x,y)) \bullet$$

$$\bullet \psi_{p(j-p)}(x,y)\exp(i2\pi\nu_j x)dxdy \,, \qquad (16)$$

where $\phi_m(x,y)$ is the DOE's phase distribution after m-th iteration.

4. Conclusion

Fundamental properties of multimode dispersionless beams have been investigated by methods of calculation and optical experiment. There is a good agreement between theory and experimental results. The dispersionless beam structure conserving feature was investigated by intensity distribution measurement in input and output planes of Fourier stage. Multimode dispersionless beam intensity investigation result showed stability of intensity structure during beam propagation. Phase structure conserving feature was investigated by digital method and interferometry. Multimode dispersionless beams fundamental properties were investigated (self-reproduction, amplitude-phase structure stability) promising good prospectives for application of such beams in future high efficient telecommunication systems.

Acknowledgements

The authors would like to thank B. Kley from Institute of Applied Physics/University Jena for manufacturing the 3D resist profile. The present research has been supported by Russian Foundation of Fundamental Research (grant number 00-15-96114).

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