

CALCULATION OF EFFECTIVE MODE FIELD AREA OF PHOTONIC CRYSTAL FIBER WITH DIGITAL IMAGE PROCESSING ALGORITHM

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Abstract

Photonic crystal fiber as a new type of optical fiber has been extensively applied because of its unique properties. The effective mode area of optical fiber is an important parameter, which has a great influence on the performance of optical fiber. In this study, digital image processing algorithm was used for preprocessing to improve the accuracy of calculation of mode field area. Then the effective mode field area of optical fiber was calculated using Matlab based Gauss fitting method. Take single-mode fiber G.652 as an example, the effective mode field area was calculated using the traditional algorithm and digital image processing algorithm respectively. It was found that the results obtained using digital image processing algorithm were within the allowed error range, suggesting the effectiveness of the algorithm. Then the calculation of the effective mode area of the triangular lattice photonic crystal fiber further verified the reliability of the algorithm.

Keywords: photonic crystal fiber, effective mode field area, image processing.

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Introduction

Photonic crystal fiber is a fast developing research field, and it has a more extensive application in the fabrication of optoelectronic devices as its performance is more superior than that of traditional optical fiber. Fu et al. [1] designed a high-voltage sensor based on photonic crystal fiber and found that the sensor had a low-temperature cross sensitivity and high-temperature sustainability, which was a potential ideal choice for downhole pressure sensing. Deng et al. [2] developed a bending sensor using photonic crystal fiber and found that the device had high bending sensitivity. Gao et al. [3] designed an all-fiber magnetic field sensor based on magnetic fluid filled photonic crystal fiber and carried out an experiment. It had a resolution of 0.09 Oe and good repeatability. Compared with other expensive methods, the method had the characteristics of high sensitivity and low cost. Effective mode field area was one of the important parameters of optical fiber, and it has a great influence on the performance of optical fiber. Measurement of effective mode field area is a very important part. It is found that the bending loss of optical fiber is small when the effective mode field area is large [4, 5]. Miyagi et al. [6] measure the effective mode area using far-field scanning and finite difference method. The difference between the values obtained by experimental measurement and numerical simulation was 0.9%~3% only. Hayashi et al. [7] studied the equation of conversion between near-field mode and far-field mode and realized effective area measurement of the higher-order modes using high-dynamic-range far-field scan technique. Mishra et al. [8] measured the effective mode area of photonic crystal fiber using full-vector finite element method. Medjouri et al. [9] measured the effective mode area of circular lattice photonic crystal fiber using finite difference time domain method in combination with the boundary condition of perfectly matched layer and found that the mode area was lar-

ger than 1000 μm^2 . In order to obtain more accurate measurement results, this study firstly performed preprocessing using digital image processing algorithm and calculated the effective mode field area of optical fiber using Matlab based Gaussian fitting method. The results showed that the results obtained were within the allowed error range, proving the effectiveness of the algorithm.

Overview of photonic crystal fiber

1. Photonic crystal fiber

Photonic crystal fiber, a new type of optical fiber, is equivalent to two-dimensional photonic crystals, which greatly promotes the development of fiber optic devices. Because of its excellent performance, photonic crystal fibers have been successfully applied in gyroscope, high-power fiber lasers and other devices.

2. Effective mode field area of optical fiber

The effective mode field area of optical fiber plays a decisive role in its nonlinear parameters and moreover is closely related to the bending loss, numerical aperture and dispersion of optical fiber [10]. It is found that photonic crystal fiber with large mode field area is more suitable for high-power applications [11, 12]. Therefore, it is of great significance to study the effective mode field area of optical fiber for the practical application of optical fiber.

At present, there are many methods to measure the effective mode field area of optical fiber, and they mainly focus calculating by mode field diameter. The main methods include far-field scanning method [13], near-field scanning method [14] and lateral displacement method. In this study, near-field scanning method was used to obtain the image of photonic crystal fiber.

There will be a large error if relevant parameters are calculated according to an ideal model as the geometric

structure of photonic crystal fiber is complex. It includes not only fibers with circular fibre core but also fibers in triangular and rectangular arrangement. Moreover the geometric structure of the interface is likely to be irregular in the process of manufacturing. The characteristics of photonic crystal fiber are easy to be affected by the external world because of its high sensitivity. During measurement, the stability of lights, sample processing quality and precision of device will all affect the characteristics of optical fiber, which brings large difficulties to the accurate measurement of mode field area. Far-field scanning technique requires high on skills, which is difficult to be realized in reality. Beam propagation method can only be realized theoretically currently. Near-field scanning method is a simple and efficient method for measuring the diameter of mode field. It can obtain a result with a small error no matter whether the core of optical fibre is circular symmetric. In order to calculate the area of mode field of optical fibre more accurately, it is necessary to preprocess the image of section of photonic crystal fiber to remove noise, reconstruct image structure and establish the numerical analysis model based on the structure. The measurement error can be reduced in this way.

Design of measurement system

Firstly, the near-field scanning method and traditional calculation method of diameter of mode field were used to calculate the area of mode field, and the result was taken as the standard. Then the image obtained by near-field scanning method was preprocessed, and the effective mode field area was calculated by Matlab based Gaussian fitting method. The structure of the measurement system is shown in Figure 1.

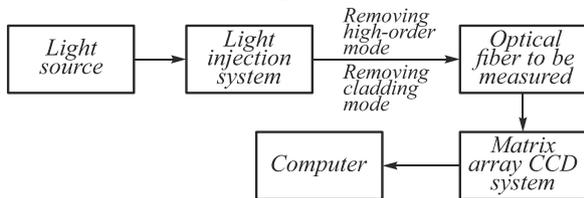


Fig. 1. The structure of near-field scanning method

- (1) Optical source. A 800 nm laser was used as the optical source in the experiment.
- (2) Light injection system. In this study, a microscope objective was combined with a collimator, and the full injection was achieved by a five-dimensional adjustment platform.
- (3) Removing high-order mode. In this study, the high-order mode was removed by means of rolling. A ring with a diameter of 3 mm wound single-mode G.652 to remove high-order mode. For triangular lattice photonic crystal fiber, two rings with a diameter of 4.5 mm was used.
- (4) Removing cladding mode. After removing the coating with a pair of optical fiber stripping forceps, it was placed in glycerol matching solution to remove the cladding mode.
- (5) The area array Charge Coupled Device (CCD) system. The array CCD was directly connected with 60X objec-

tive lens, and the infrared CCD was selected. Besides, the resolution could be improved by replacing eye lens with CCD. The CCD system could convert the acquired light intensity signal to electrical signals. The distance d between two adjacent pixels of the CCD could be determined by the following formula: $\sigma \times \Gamma \geq 2d$, where σ represents the resolution of the microscope and represents the magnifying power of optical vision.

The resolution of the eye lens of the CCD used in this study was $0.57 \mu\text{m}$, and the magnifying power was 60; then the pixel spacing of CCD was:

$$d \leq \frac{\sigma \times \Gamma}{2} = \frac{0.62 \times 60}{2} = 18.6 \mu\text{m}$$

Digital imaging processing algorithm

Preprocessing of photonic crystal fiber image

(1) Magnification processing of image

In order to improve the accuracy of image recognition, it was necessary to enlarge the cross section image of photonic crystal fiber image obtained by scanning electron microscope. The image was magnified two times using three times of convolution interpolation (Figure 2).

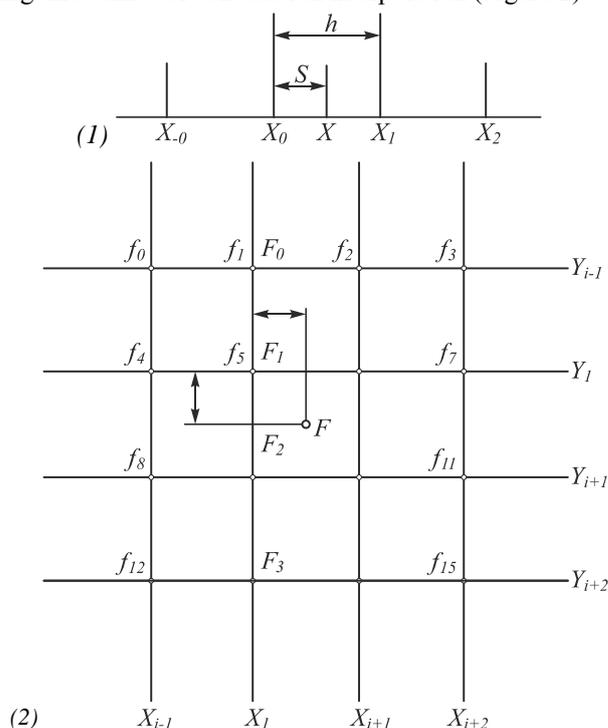


Fig. 2. The schematic diagram of magnification using three times of convolution

f is used for presenting a sampling function. Through interpolation, g is used to approximate f . Then

$$g(x) = \sum_k c_k u((x - x_k)/h), \tag{1}$$

where x_k refers to node of interpolation, u refers to interpolation kernel, h refers to sampling interval (as shown in Figure 2), and c_k is a parameter.

The kernel function of the three convolution interpolations are:

$$u(s) = \begin{cases} \frac{3}{2}|s|^3 - \frac{5}{2}|s|^2 + 1, & 0 < |s| < 1, \\ -\frac{1}{2}|s|^3 + \frac{5}{2}|s|^2 - 4|s| + 2, & 1 < |s| < 2, \\ 0, & 2 < |s|. \end{cases} \quad (2)$$

(2) Medium filtering denoising

The noise appeared in image acquisition process has a great influence on the quality and accuracy of images, and noise may be magnified after images are magnified, which can affect image information. Therefore it is necessary to do denoising. Median filter denoising method can not only effectively remove noise, but also protect the edge information of images. Wiener filtering is a kind of filter with favourable performance. The original image was represented by $f(x, y)$, and the output image was represented by $\hat{f}(x, y)$. The computational formula of mean square error e^2 is:

$$e^2 = E\{[f - \hat{f}]^2\}. \quad (3)$$

After deduction, it turns to be:

$$\hat{F}(u, v) = \left[\frac{1}{H(u, v)} \frac{|H(u, v)|^2}{|H(u, v)|^2 + K} \right] G(u, v). \quad (4)$$

where $H(x, y)$ refers to the Fourier transformation of the degenerate function, $G(x, y)$ refers to the Fourier transformation of the original image, and K refers to the ratio of power density of noise to signal.

The median of field brightness protects the edge information of image in addition to removing impact noise, and its expression is:

$$m = Med\{x_1, x_2, x_3, \dots, x_n\} = \begin{cases} x_{i((n+1)/2)}, & n \text{ is odd number,} \\ \frac{1}{2}[x_{i(n/2)} + x_{i((n/2)+1)}], & n \text{ is even number.} \end{cases} \quad (5)$$

Non-linear filtering technology was used for replacing the pixel gray value of specified points with the median value of gray values of different points. The input sequence is $\{x_i, i \in I\}$, where I stands for natural number. The length of window was l .

$$m_i = Med\{x_i\} = Med\{x_{i-u}, \dots, x_i, \dots, x_{i+u}\}. \quad (6)$$

Median filtering was calculated using two-dimensional window. $\{x_{ij}, (i, j) \in I^2\}$ represents the gray value of each point in the digital image. Filtering window was H .

$$y_{ij} = Med_H\{x_{ij}\} = Med\{x_{i+r, j+s} | (r, s) \in H, (i, j) \in I^2\}. \quad (7)$$

Taking a photonic crystal fiber image as an example, Wiener filtering and median filtering were used to denoise the image (Figure 3).

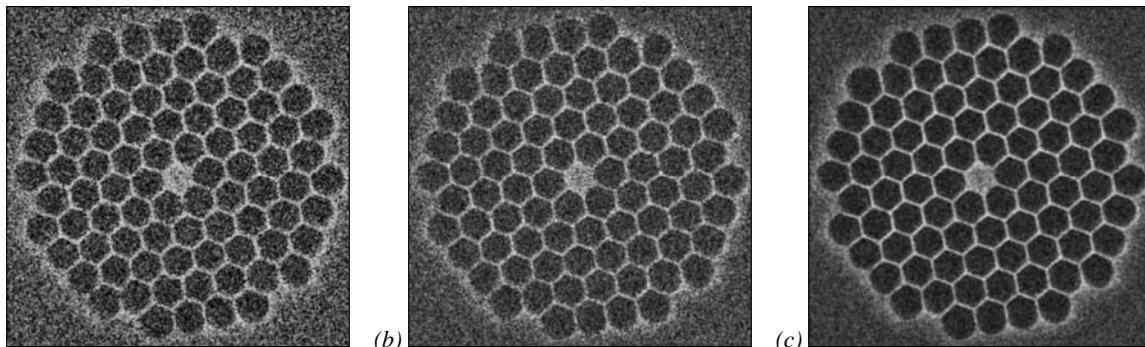


Fig. 3. Comparison of denoising effects: (a) original image; (b) image after Wiener filtering; (c) image after median filtering

It could be seen from Figure 3 that the denoising effect of median filtering was superior to that of Wiener filtering. Median filtering was excellent in denoising images and reduce the effects of noise on images.

(3) Otsu binarization

Performing binarization on the image with proper threshold was to separate air vent holes and media in optical fiber. Otsu algorithm is an ideal algorithm. The segmentation threshold of foreground and background was set as T ; the proportion of points of foreground was set as w_0 , and the average gray level was set as u_0 ; the proportion of points of background was set as w_1 , and the average gray level was set as u_1 ; the total average gray level was u ; the variance of the foreground and background images was g . Then

$$u = w_0 \times u_0 + w_1 \times u_1, \quad (8)$$

$$g = w_0 \times (u_0 - u)^2 + w_1 \times (u_1 - u)^2.$$

The following equation is obtained by considering the above two equations:

$$g = \frac{w_0}{1 - w_0} \times (u_0 - u)^2. \quad (9)$$

When variance g was the largest, gray level T was the optimal threshold. Suppose the scanned image of photonic crystal fibre as $f(x, y)$, then the binary image after segmentation is:

$$h(x, y) = \begin{cases} 1, & f(x, y) \geq T, \\ 0, & f(x, y) < T. \end{cases} \quad (10)$$

1. Calculation of effective mode field area

$$A_{eff} = \frac{\left(\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |E(x, y)|^2 dx dy \right)^2}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |E(x, y)|^4 dx dy}, \tag{11}$$

where $E(x, y)$ stands for the transverse electrical field component of basic mode on the horizontal section of the optical fibre.

The following equation can be obtained by substituting the cross section area s into equation (11):

$$A_{eff} = \frac{\left(\int_s \int_s |E(x, y)|^2 dx dy \right)^2}{\int_s \int_s |E(x, y)|^4 dx dy}. \tag{12}$$

As electric field intensity is related to light intensity: $I(x, y) = |E(x, y)|^2$, where $I(x, y)$ stands for the light intensity component of the basic mode of the optical fiber interface. Equation (13) is obtained according to equation (12).

$$A_{eff} = \frac{\left(\int_s \int_s |I(x, y)| dx dy \right)^2}{\int_s \int_s |I(x, y)|^2 dx dy} \tag{13}$$

If the light intensity distribution of basic mode satisfies approximate Gaussian distribution, then equation (13) can be simplified as:

$$A_{eff} = \pi\omega^2. \tag{14}$$

The effective mode field area was calculated using Matlab based Gaussian fitting method. The image was obtained from the CCD acquisition system. Light spot image whose pixel value was between 235 and 255 was saved. Then the image was read using Microsoft foundation classes and preprocessed. Then data of radial pixel points were obtained through Matlab, and a two-dimensional Gaussian curve was drawn. Finally the effective mode field area was calculated using equation (14).

Simulation and results

To verify the accuracy of the above algorithm, single-mode optical fiber G.652 and triangular lattice photonic crystal fiber were taken as examples.

Single-mode optical fiber

Firstly the diameter of mode field of the optical fiber was measured using the traditional method, and then the effective mode field area was calculated; the result was taken as the standard and compared to the result obtained by digital image processing algorithm. A 800 nm laser was used as the light source.

The official data of single-mode optical fiber G.652 suggested that the diameter of mode field of the optical fiber was $9.2 \pm 0.4 \mu\text{m}$ and $10.4 \pm 0.5 \mu\text{m}$ respectively under 1310 nm and 1550 nm wavelength. As the mode field area was in a linear relationship with the incident light wave, the effective mode field area of the optical fiber under 800 nm wavelength could be calculated, $27.2 \mu\text{m}^2$.

The effective mode field area was also calculated using digital image processing algorithm. Firstly the image of the light spot was obtained using near-field scanning method, as shown in Figure 4. Median filtering denoising and binarization were performed after double amplification with cubic convolution interpolation. The data of radial pixel points were obtained through Matlab. Then a two-dimensional Gaussian curve was drawn. Finally the effective mode field area of the the optical fiber under 800 nm wavelength was calculated, $28.4 \mu\text{m}^2$.

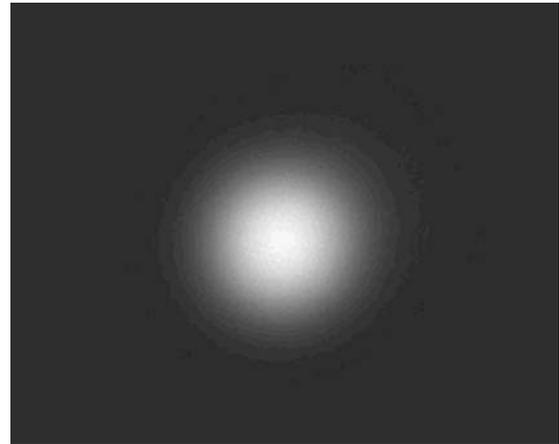


Fig. 4. The light spot image of the single-mode optical fiber

According to the official data, the diameter d_1 of the mode field under the wavelength of 1310 nm was $9.2 \mu\text{m}$, and the error α_1 was $\pm 0.4 \mu\text{m}$; the diameter d_2 of the mode field under the wavelength of 1550 nm was $10.4 \mu\text{m}$, and the error α_2 was $\pm 0.5 \mu\text{m}$. The relative error of the mode field area was calculated using the following equation.

$$\alpha = \left| \frac{A_{eff1} - A_{eff2}}{A_{eff1}} \right| \times 100\% = \left| \frac{\pi(d_1/2)^2 - \pi((d_1 + \alpha_1)/2)^2}{\pi(d_1/2)^2} \right| \times 100\%. \tag{15}$$

The error of the mode field area of the optical fiber was 9% and 9.8% under the wavelength of 1310 nm and 1550 nm.

Using the algorithm, the effective mode field area under the wavelength of 800 nm was $28.4 \mu\text{m}^2$, and its relative error with the standard result, $27.2 \mu\text{m}^2$, was 4.4%, which was within the error range. Therefore the result was accurate.

Triangular lattice photonic crystal fiber

The image acquired by near-field scanning method is shown in Figure 5.

The maximum value of pixel of the image and the radial pixel value of the light spot were output using the method which was the same with the single-mode optical fiber. After read by Matlab, a Gaussian curve was drawn for Gaussian fitting. The horizontal coordinate value was read when the light intensity was the largest. Finally the effective mode field area was calculated using equation

(14). The effective mode field area of the optical fiber was $28.8 \mu\text{m}^2$ under the wavelength of 800 nm.

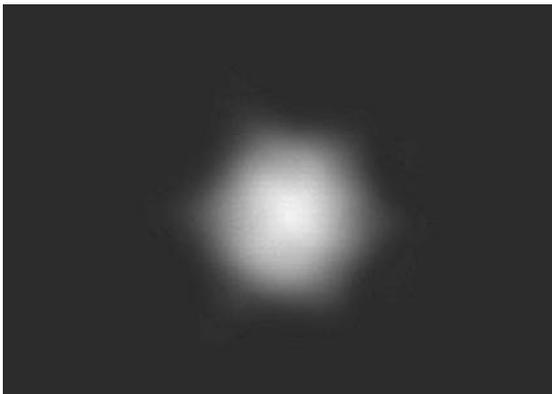


Fig. 5. The light spot image of triangular lattice photonic crystal fiber

The mode field area was measured based on the diameter. Firstly the mode field diameter of the horizontal light intensity distribution curve of the light spot was obtained, $6.12 \mu\text{m}$; then the mode field area was calculated, $28.6 \mu\text{m}^2$. The vertical mode field diameter of the light spot was calculated using the same method, and the result was $6.57 \mu\text{m}$; the mode field area was $30.1 \mu\text{m}^2$. Therefore the average value of the effective mode field area was $29.4 \mu\text{m}^2$. The results obtained by the two algorithms were close and within the allowed error range, suggesting the high accuracy of digital image processing algorithm.

Discussion and conclusion

Photonic crystal fiber has many advantages, which greatly promotes the development of optoelectronic devices. It shows good performance on many devices. For example, the magnetic field sensor which is made from photonic crystal fiber has superior sensitivity [15]; temperature sensor based on photonic crystal fiber has not only a higher temperature sensitivity but also enhanced refractive index sensitivity [16,17]; salinity sensors which apply photonic crystal fibers also has high sensitivity [18]; the performance of hydrostatic pressure sensors based on photonic crystal fiber is excellent [19]. Effective mode field area is one of the important parameters of optical fiber, which plays a decisive role in the nonlinear coefficient of optical fiber and moreover has a great influence on the bending loss and weld loss of light. Therefore, the study of the effective mode field area of optical fiber plays a very important role in practical applications.

For most optical fibers with circular cores, mode field diameter is often used to describe the distribution characteristics of mode field. However, for many non-circular-symmetrical optical fibers, it is necessary to measure effective mode field area as mode field diameter cannot be accurately obtained.

The differences of grayscale and contrast between images collected by near-field scanning method bring great difficulties to measurement and calculation. In order to eliminate these effects, image pre-processing is needed. The accuracy of measurement will be improved by image magnification, denoising and binarization. After image

preprocessing, the effective mode field area of optical fiber can be obtained by Matlab based Gaussian fitting method.

In order to verify the effectiveness of the algorithm, the single-mode optical fiber G.652 was calculated in two ways, and the calculation results showed that the area of the effective mode field was within the allowed error range, and the correctness of the algorithm was also proved. Then the effective mode field area of a triangular lattice photonic crystal fiber was calculated, which further verified the correctness of the algorithm.

The experiment showed that digital image processing algorithm had great practicability in calculating the effective mode field area of the photonic crystal fiber with non-circular core and it improved the accuracy of the image, reduce the complexity of the calculation, and reduce the measurement error. Image processing algorithms are of great value in the measurement research of irregular optical fibers.

References

- [1] Fu HY, Wu C, Tse MLV, Zhang L, Cheng KCD, Tam HY, Guan BO, Lu C. High pressure sensor based on photonic crystal fiber for downhole application. *Appl Opt* 2010; 49(14): 2639-2643. DOI: 10.1364/AO.49.002639.
- [2] Deng M, Tang C-P, Zhu T, Rao Y-J. Highly sensitive bend sensor based on Mach-Zehnder interferometer using photonic crystal fiber. *Opt Commun* 2011; 284(12): 2849-2853. DOI: 10.1016/j.optcom.2011.02.061.
- [3] Gao R, Jiang Y, Abdelaziz S. All-fiber magnetic field sensors based on magnetic fluid-filled photonic crystal fibers. *Opt Lett* 2013; 38(9): 1539-1541. DOI: 10.1364/OL.38.001539.
- [4] Matsui T, Sakamoto T, Tsujikawa K, Tomita S, Tsubokawa M. Single-mode photonic crystal fiber design with ultralarge effective area and low bending loss for ultrahigh-speed WDM transmission. *J Lightw Technol* 2011; 29(4): 511-515. DOI: 10.1109/JLT.2010.2089600.
- [5] Napierała M, Nasilowski T, Bereś-Pawlik E, Mergo P, Berghmans F, Thienpont H. Large-mode-area photonic crystal fiber with double lattice constant structure and low bending loss. *Opt Express* 2011; 19(23): 22628-22636. DOI: 10.1364/OE.19.022628.
- [6] Miyagi K, Namihira Y, Razzak SMA, Kaijage SF, Begum F. Measurements of mode field diameter and effective area for photonic crystal fibers by far field scanning technique. *Optical Review* 2010; 17(4): 388-392. DOI: 10.1007/s10043-010-0072-x.
- [7] Hayashi T, Tamura Y, Nagashima T, Yonezawa K, Taru T, Igarashi K, Soma D, Wakayama Y, Tsuritani T. Effective area measurement of few-mode fiber using far field scan technique with Hankel transform generalized for circularly-asymmetric mode. *Opt Express* 2018; 26(9): 11137-11146. DOI: 10.1364/OE.26.011137.
- [8] Mishra SS, Singh VK. Polarization maintaining highly birefringent small mode area photonic crystal fiber at telecommunication window. *J Microw Optoelectron Electromagn Appl* 2011; 10(1): 33-41. DOI: 10.1590/S2179-10742011000100004.
- [9] Medjouri A, Simohamed LM, Ziane O, Boudrioua A. Analysis of a new circular photonic crystal fiber with large mode area. *Optik* 2015; 126(24): 5718-5724. DOI: 10.1016/j.ijleo.2015.09.035.
- [10] Abdelaziz I, Abdelmalek F, Ademgil H, Haxha S, Gorman T, Bouchriha H. Enhanced effective area photonic crystal

- fiber with novel air hole design. *J Lightw Technol* 2010; 28(19): 2810-2817. DOI: 10.1109/JLT.2010.2064758.
- [11] Saini TS, Kumar A, Sinha RK. Triangular-core large-mode-area photonic crystal fiber with low bending loss for high power applications. *Appl Opt* 2014; 53(31): 7246-7251. DOI: 10.1364/AO.53.007246.
- [12] Liu Y, Dong X, Liu Z, Sun B, Ji J, Yu X. Splicing and end facet optimization of large-mode-area photonic crystal fiber for high power application. 2017 16th International Conference on Optical Communications and Networks (ICOON) 2017: 1-3. DOI: 10.1109/ICOON.2017.8121275.
- [13] Miyagi K, Namihira Y, Kasamatsu Y, Hossain MA. Dynamic control of mode field diameter and effective area by germanium doping of hexagonal photonic crystal fibers. *Optical Review* 2013; 20(4): 327-331. DOI: 10.1007/s10043-013-0059-5.
- [14] Filipenko O, Sychova O, Ponomaryova G. Determining of the photonic-crystal fibers mode field size at his near field image. Third International Scientific-Practical Conference Problems of Infocommunications Science and Technology (PIC S&T) 2017: 81-83. DOI: 10.1109/INFOCOMMST.2016.7905342.
- [15] Thakur HV, Nalawade SM, Gupta S, Kitture R, Kale SN. Photonic crystal fiber injected with Fe₃O₄ nanofluid for magnetic field detection. *Appl Phys Lett* 2011; 99(16): 161101. DOI: 10.1063/1.3651490.
- [16] Yu Y, Li X, Hong X, Deng Y, Song K, Geng Y, Wei H, Tong W. Some features of the photonic crystal fiber temperature sensor with liquid ethanol filling. *Opt Express* 2010; 18(15): 15383-15388. DOI: 10.1364/OE.18.015383.
- [17] Qiu S-J, Chen Y, Xu F, Lu Y-Q. Temperature sensor based on an isopropanol-sealed photonic crystal fiber in-line interferometer with enhanced refractive index sensitivity. *Opt Lett* 2012; 37(5): 863-865. DOI: 10.1364/OL.37.000863.
- [18] Vigneswaran D, Ayyanar N, Sharma M, Sumathi M, Rajan MSM, Porsezian K. Salinity sensor using photonic crystal fiber. *Sensors and Actuators A: Physical* 2018; 269: 22-28. DOI: 10.1016/j.sna.2017.10.052.
- [19] Li H-T, Wang X-L, She L-J, Chen D-R. Dual-core photonic crystal fiber for hydrostatic pressure sensing. *Acta Photonica Sinica* 2017; 46(7): 0706007. DOI: 10.3788/gzxb20174607.0706007.

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